

# Scalar Mesons in QCD \*

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We review the analysis of the quark and gluon substructures of the scalar mesons from QCD spectral sum rules and some low-energy theorems applied to the scalar QCD anomaly current. The present data favour equal components of  $\bar{u}u + \bar{d}d$  and of  $gg$  in the wave functions of the low-mass (below 1 GeV) scalar mesons, which make the wide  $\sigma$  and the narrow  $f_0(980)$  as  $\eta'$ -like particles, which can have strong couplings to meson pairs through OZI violations. A coherent picture of the other  $I = 0$  scalar mesons spectra within this mixing scheme is shortly discussed. We also expect the  $a_0(980)$  to be the lowest isovector  $\bar{u}d$  state, and the  $K_0^*(1430)$  its  $\bar{d}s$  partner.

## 1. INTRODUCTION

Since the discovery of QCD, it has been emphasized [1] that exotic mesons beyond the standard octet, exist as a consequence of the non-perturbative aspects of quantum chromodynamics (QCD). Among these states, the  $\eta'$  meson is a *peculiar* pseudoscalar state, as it should contain a large gluon component (the 1st well-established gluonium!) in its wave function for explaining why its mass is not degenerate with the one of the pion but only vanishes in the large  $N_c$  limit (solution of the so-called  $U(1)_A$  problem) [2], though the  $\eta'$  likes to couple to  $\bar{q}q$  mesons in its decay (e.g.  $\eta' \rightarrow \pi\pi\pi, \dots$ ). This peculiar feature has led to certain confusion in the interpretation of its nature. Even, at present, many physicists claim misleadingly that the  $\eta'$  is a  $\bar{q}q$  state, from the conclusion based on the successful quark model prediction of its hadronic and  $2\gamma$  couplings. This feature is not too surprising due to the manifestation of the strong OZI-violation for low mass state in this channel. However, the  $\eta'$  mass is lower than the direct calculation of the pseudoscalar glueball mass from QCD spectral sum rules (QSSR) and lattice QCD (see Table 1). This apparent discrepancy can be understood from the crucial rôle played by the  $\eta'$  in the evaluation of the  $U(1)_A$  topological susceptibility and of its

slope [3]. In the effective Lagrangian approach, it has been expected since the pioneer work of Nambu-Lonasinio that an analogue of the pion of the non-linear  $\sigma$  model ( $NL\sigma M$ ) exists in the linear  $\sigma$  model ( $L\sigma M$ ) [4], but this  $L\sigma M$  has not attracted too much attention in the past because it has been observed that the  $SU(2)_L \times SU(2)_R$  chiral symmetry of QCD is non-linearly realized (this has made the success of the chiral perturbation theory (ChPT) approach [5]). It also occurs that the Lagrangian of the  $L\sigma M$  is not unique such that no definite predictions from QCD first principles can be done. In addition, the inclusion of the resonances into the effective Lagrangian is not completely settled [6].

However, this analogy between the pion and the  $\sigma$  meson may not be very appropriate as they are particles associated to symmetries of a different nature ( $SU(2)_L \times SU(2)_R$  for the pion and  $U(1)_V$  for the  $\sigma$ ). Instead, a comparison of the  $\sigma$  with the  $\eta'$  meson looks more valuable within a chiral  $U(3)_A \times U(3)_V$  Lagrangian [2,7], as both particles are associated respectively to the  $U(1)$  axial and vector symmetries, where in terms of the quark and gluon fields, the corresponding QCD currents are respectively, the  $U(1)_A$  anomaly (divergence of the singlet axial current):

$$\begin{aligned} \partial_\mu A^\mu(x) &= \left( \frac{\alpha_s}{8\pi} \right) \text{tr} G_{\alpha\beta} \tilde{G}^{\alpha\beta} \\ &+ \sum_{u,d,s} m_q \bar{q}(i\gamma_5)q, \end{aligned} \quad (1)$$

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and the scalar anomaly dilaton current:

$$\theta_\mu^\mu = \frac{1}{4}\beta(\alpha_s)G_{\alpha\beta}G^{\alpha\beta} + (1 + \gamma_m(\alpha_s)) \sum_{u,d,s} m_q \bar{q}q, \quad (2)$$

which is the trace of the energy-momentum tensor  $\theta_{\mu\nu}$ . The sum over colour is understood;  $q$ ,  $G_{\mu\nu}$  and  $\tilde{G}_{\mu\nu}$  are respectively the quark, the gluon field strength and its dual;  $m_q$  is the light quark running mass;  $\beta$  and  $\gamma_m$  are respectively the  $\beta$ -function and quark mass anomalous dimension.

In this talk, we shall discuss the  $\sigma$  meson from this point of view of the analogy with the  $\eta'$ -meson by using QCD spectral sum rules (QSSR) à la SVZ [8] (for a review, see e.g.: [9]) and some low-energy theorems based on Ward identities. The discussions are based on the works in [10–12].<sup>2</sup>

## 2. AN OUTLINE OF QCD SPECTRAL SUM RULES (QSSR)

### 2.1. Description of the method

Since its discovery in 79, QSSR has proved to be a powerful method for understanding the hadronic properties in terms of the fundamental QCD parameters such as the QCD coupling  $\alpha_s$ , the (running) quark masses and the quark and/or gluon QCD vacuum condensates. The description of the method has been often discussed in the literature, where a pedagogical introduction can be, for instance, found in the book [9]. In practice (like also the lattice), one starts the analysis from the two-point correlator:

$$\psi_H(q^2) \equiv i \int d^4x e^{iqx} \langle 0 | \mathcal{T} J_H(x) (J_H(0))^\dagger | 0 \rangle, \quad (3)$$

built from the hadronic local currents  $J_H(x)$ , which select some specific quantum numbers. However, unlike the lattice which evaluates the correlator in the Minkowski space-time, one exploits, in the sum rule approaches, the analyticity property of the correlator which obeys the well-known Källén–Lehmann dispersion relation:

$$\psi_H(q^2) = \int_0^\infty \frac{dt}{t - q^2 - i\epsilon} \frac{1}{\pi} \text{Im}\psi_H(t) + \dots, \quad (4)$$

<sup>2</sup>Discussions of the  $\bar{q}q$  and/or gluonium nature of scalar states can also be found in e.g.[9],[13]–[23].

where ... represent subtraction points, which are polynomials in the  $q^2$ -variable. In this way, the *sum rule* expresses in a clear way the *duality* between the integral involving the spectral function  $\text{Im}\psi_H(t)$  (which can be measured experimentally), and the full correlator  $\psi_H(q^2)$ . The latter can be calculated directly in the QCD Euclidean space-time using perturbation theory (provided that  $-q^2 + m^2$  ( $m$  being the quark mass) is much greater than  $\Lambda^2$ ), and the Wilson expansion in terms of the increasing dimensions of the quark and/or gluon condensates which simulate the non-perturbative effects of QCD.

### 2.2. Beyond the usual SVZ expansion

Using the Operator Product Expansion (OPE) [8], the two-point correlator reads:

$$\psi_H(q^2) \simeq \sum_{D=0,2,4,\dots} \frac{1}{(-q^2)^{D/2}} \times \sum_{\dim O=D} C(q^2, \nu) \langle \mathcal{O}(\nu) \rangle, \quad (5)$$

where  $\nu$  is an arbitrary scale that separates the long- and short-distance dynamics;  $C$  are the Wilson coefficients calculable in perturbative QCD by means of Feynman diagrams techniques;  $\langle \mathcal{O}(\nu) \rangle$  are the quark and/or gluon condensates of dimension  $D$ . In this paper, we work in the massless quark limit. Then, one may expect the absence of the terms of dimension 2 due to gauge invariance. However, it has been emphasized recently [24] that the resummation of the large order terms of the perturbative series, and the effects of the higher dimension condensates due e.g. to instantons, can be mimicked by the effect of a tachyonic gluon mass, which might be understood from the short distance linear part of the QCD potential. The strength of this short distance mass has been estimated from the  $e^+e^-$  data to be [25,26]:  $\frac{\alpha_s}{\pi}\lambda^2 \simeq -(0.06 \sim 0.07) \text{ GeV}^2$ , which leads to the value of the square of the (short distance) string tension:  $\sigma \simeq -\frac{2}{3}\alpha_s\lambda^2 \simeq [(400 \pm 20) \text{ MeV}]^2$  in an (unexpected) good agreement with the lattice result [27] of about  $[(440 \pm 38) \text{ MeV}]^2$ . The strengths of the vacuum condensates having dimensions  $D \leq 6$  are also under good control: namely  $2m\langle \bar{q}q \rangle = -m_\pi^2 f_\pi^2$  from pion PCAC,  $\langle \alpha_s G^2 \rangle = (0.07 \pm 0.01) \text{ GeV}^2$  from

$e^+e^- \rightarrow I = 1$  data [25] and from the heavy quark mass-splittings [28],  $\alpha_s \langle \bar{q}q \rangle^2 \simeq 5.8 \times 10^{-4} \text{ GeV}^6$  [25], and  $g^3 \langle G^3 \rangle \approx 1.2 \text{ GeV}^2 \langle \alpha_s G^2 \rangle$  from dilute gaz instantons [13].

### 2.3. Spectral function

In the absence of complete data, the spectral function is often parametrized using the “naïve” duality ansatz:

$$\frac{1}{\pi} \text{Im}\psi_H(t) \simeq 2M_H^{2n} f_H^2 \delta(t - M_H^2) + \text{“QCD continuum”} \times \theta(t - t_c), \quad (6)$$

which has been tested [9] using  $e^+e^-$ ,  $\tau$ -decay data, to give a good description of the spectral integral in the sum rule analysis;  $f_H$  (analogue to  $f_\pi$ ) is the hadron’s coupling to the current;  $2n$  is the dimension of the correlator, while  $\sqrt{t_c}$  is the QCD continuum’s threshold.

### 2.4. Form of the sum rules and optimization procedure

Among the different sum rules discussed in the literature within QCD [9] (Finite Energy Sum rule (FESR) [29],  $\tau$ -like sum rules [30],...), we shall mainly be concerned here with:

- The exponential Laplace unsubtracted sum rule (USR) and its ratio:

$$\begin{aligned} \mathcal{L}_n(\tau) &= \int_0^\infty dt t^n \exp(-t\tau) \frac{1}{\pi} \text{Im}\psi_H(t), \\ \mathcal{R}_n &\equiv -\frac{d}{d\tau} \log \mathcal{L}_n, \quad (n \geq 0); \end{aligned} \quad (7)$$

- The subtracted sum rule (SSR):

$$\begin{aligned} \mathcal{L}_{-1}(\tau) &= \int_0^\infty \frac{dt}{t} \exp(-t\tau) \frac{1}{\pi} \text{Im}\psi_H(t) \\ &\quad + \psi_H(0). \end{aligned} \quad (8)$$

The advantage of the Laplace sum rules with respect to the previous dispersion relation is the presence of the exponential weight factor, which enhances the contribution of the lowest resonance and low-energy region accessible experimentally. For the QCD side, this procedure has eliminated the ambiguity carried by subtraction constants, arbitrary polynomial in  $q^2$ , and has improved the convergence of the OPE by the presence of the

factorial dumping factor for each condensates of given dimensions. The ratio of the sum rules is a useful quantity to work with, in the determination of the resonance mass, as it is equal to the meson mass squared, in the usual duality ansatz parametrization. As one can notice, there are “a priori” two free external parameters ( $\tau, t_c$ ) in the analysis. The optimized result will be (in principle) insensitive to their variations. In some cases, the  $t_c$ -stability is not reached due to the too naïve parametrization of the spectral function. One can either fixed the  $t_c$ -values by the help of FESR (local duality) or improve the parametrization of the spectral function by introducing threshold effects fixed by chiral perturbation theory, ..., in order to restore the  $t_c$ -stability of the results. The results discussed below satisfy these stability criteria.

## 3. UNMIXED GLUONIA MASSES AND DECAY CONSTANTS

Before discussing the specific scalar channel, let’s present the situation of gluonia/glueball mass calculations as a guide for the forthcoming discussions.

### 3.1. The currents

In addition to the pseudoscalar and scalar currents introduced previously, we shall deal with the tensor and 3-gluon currents (standard notations):

$$\begin{aligned} \theta_{\mu\nu} &= -G_\mu^\alpha G_{\nu\alpha} + \frac{1}{4} g_{\mu\nu} G^2, \\ J_3 &= g f_{abc} G_{\alpha\beta}^a G_{\beta,\gamma}^b G_{\gamma\alpha}^c, \end{aligned} \quad (9)$$

### 3.2. Masses and decay constants

The unmixed gluonia masses from the unsubtracted QCD Spectral Sum Rules (USR) [12] are compared in Table 1 with the ones from the lattice [27,31] in the quenched approximation, where we use the conservative guessed estimate of about 15% for the different lattice systematic errors (separation of the lowest ground states from the radial excitations, which are expected to be nearby as indicated by the sum rule analysis; discretisation; quenched approximation,...). One can notice an excellent agreement between the USR and the lattice results, with the mass hierarchy:  $M_{0++} \leq M_{0-+} \approx M_{2++}$ , expected from

Table 1

Unmixed gluonia masses and decay constants from QSSR [12] compared with the lattice.

$J^{PC}$	Name	Mass [GeV]			$\sqrt{t_c}$ [GeV]	$f_H$ [MeV]
		Estimate	Upper Bound	Lattice [27,31]		
$0^{++}$	$G$	$1.5 \pm 0.2$	$2.16 \pm 0.22$	$1.60 \pm 0.16$	2.1	$390 \pm 145$
	$3G$	3.1	3.7		3.4	62
$2^{++}$	$T$	$2.0 \pm 0.1$	$2.7 \pm 0.4$	$2.26 \pm 0.22$	2.2	$80 \pm 14$
$0^{-+}$	$P$	$2.05 \pm 0.19$	$2.34 \pm 0.42$	$2.19 \pm 0.32$	2.2	$8 \sim 17$

some QCD inequalities [32]. However, this is not the whole story ! Indeed, one can notice that in the pseudoscalar channel, the predicted value of the mass of the  $0^{-+}$  is too high compared with the mass of the  $\eta'$ , which is not surprising for the reasons explained in the introduction. We shall see in the next section that the same phenomena occur for the scalar channel.

#### 4. UNMIXED SCALAR GLUONIA

##### 4.1. The need for a low mass $\sigma_B$ from the sum rules

Using the mass and decay constant of the scalar gluonium  $G$  in Table 1 from the USR (Eq.7), into the SSR (Eq.8) sum rules, where [13]  $\psi_s(0) \simeq -16(\beta_1/\pi)\langle\alpha_s G^2\rangle$ , one can notice [10,12] that one needs a second resonance <sup>3</sup> with a lower mass (the  $\sigma_B$ ) for a consistency of the two sum rules <sup>4</sup>. Using, e.g.,  $M_{\sigma_B} \simeq 1$  GeV <sup>5</sup>, one gets [10,12]:  $f_{\sigma_B} \approx 1$  GeV, which is larger than  $f_G \simeq .4$  GeV.

##### 4.2. Low-energy theorems (LET) for the couplings to meson pairs

In order to estimate the couplings of the gluonium to meson pairs, we use some sets of low-energy theorems (LET) based on Ward identities for the

<sup>3</sup>All previous old sum rule analysis based on a one-resonance parametrization [33] lead to inconsistencies, as emphasized many years ago in [10,12].

<sup>4</sup>The introduction of the  $1/q^2$  in the OPE has been also shown to restore the mass hierarchy between the gluonium and meson scales [26], and then improve further the conclusion reached here. A recent analysis using gaussian sum rules and instanton approach also favours a two-resonance parametrization of the spectral function [34].

<sup>5</sup>We cannot fix both the mass and decay constant of the  $\sigma$  from the two sum rules. We give in the original papers a more complete analysis for different values of the  $\sigma$  mass ranging from 500 to 1 GeV.

vertex:

$$\begin{aligned}
 V(q^2 \equiv (p-p')^2 = 0) &\equiv \langle H(p) | \theta_\mu^\mu | H(p') \rangle \\
 &\simeq 2m_H^2, \\
 V'(0) &= 1,
 \end{aligned} \tag{10}$$

and write the vertex in a dispersive form.  $H$  can be a Goldstone boson ( $\pi, K, \eta_8$ ), a  $\eta_1$ -  $U(1)_A$ -singlet, or a  $\sigma_B$ . Then, one obtains the sum rules for the hadronic couplings:

$$\begin{aligned}
 \frac{1}{4} \sum_{\sigma_B, \sigma'_B, G} g_{SHH} \sqrt{2} f_S &\approx 2M_H^2, \\
 \frac{1}{4} \sum_{\sigma_B, \sigma'_B, G} g_{SHH} \sqrt{2} f_S / M_S^2 &\simeq 1.
 \end{aligned} \tag{11}$$

• **Decays into  $\pi\pi$ :** Neglecting, to a first approximation the  $G$ -contribution, the  $\sigma_B$  and  $\sigma'_B$  widths to  $\pi\pi$ ,  $KK, \dots$  (we take  $M_{\sigma'} \approx 1.37$  GeV as an illustration) are [12] <sup>6</sup>:

$$\begin{aligned}
 \Gamma(\sigma_B \rightarrow \pi\pi) &\approx 0.8 \text{ GeV}, \\
 \Gamma(\sigma'_B \rightarrow \pi\pi) &\approx 2 \text{ GeV},
 \end{aligned} \tag{12}$$

which suggests a huge OZI violation and seriously questions the validity of the lattice results in the quenched approximation. Similar conclusions have been reached in [15–18]. For testing the above result, one should evaluate on the lattice, the decay mixing 3-point function  $V(0)$  responsible for such decays using dynamical fermions.

• **Decays into  $\eta'\eta$  and  $\eta\eta$ :** Using  $\eta' \approx \cos\theta_P \eta_1$  and  $\eta \approx \sin\theta_P \eta_1$ , where  $\theta_P$  is the pseudoscalar

<sup>6</sup>However, one can notice that the  $\sigma$  coupling to pion pairs decreases like  $1/f_\sigma$ , i.e., a too low value of the mass say less than 500 MeV, leads to a width less than 100 MeV. This feature does not favour a too low value of the  $\sigma$  mass and questions the interpretation of some data.

mixing angle, the previous LET implies the *characteristic gluonium decay* (we use  $M_G \approx 1.5$  GeV and assume a G-dominance in the sum rule) [10,12]:

$$\begin{aligned} \Gamma(G \rightarrow \eta\eta') &\approx (5 - 10) \text{ MeV} , \\ \frac{\Gamma(G \rightarrow \eta\eta)}{\Gamma(G \rightarrow \eta\eta')} &\approx 0.22 : \\ g_{G\eta\eta} &\simeq \sin\theta_P g_{G\eta\eta'} . \end{aligned} \quad (13)$$

• **Decay into  $4\pi^0$ :** Assuming that the  $G$  decay into  $4\pi^0$  occurs through  $\sigma_B\sigma_B$ , and using the data for  $f_0(1.37) \rightarrow (4\pi^0)_S$ , one obtains from the previous sum rule [10,12]:

$$\Gamma(G \rightarrow \sigma_B\sigma_B \rightarrow 4\pi) \approx (60 - 140) \text{ MeV} . \quad (14)$$

#### 4.3. $\gamma\gamma$ widths and $J/\psi \rightarrow \gamma S$ radiative decays

These widths can be estimated from the quark box or anomaly diagrams [10,12]. The  $\gamma\gamma$  widths of the  $\sigma, \sigma'$  and  $G$  are much smaller (factor 2 to 5) than  $\Gamma(\eta' \rightarrow \gamma\gamma) \simeq 4 \text{ keV}$ , while  $B(J/\psi \rightarrow \gamma \sigma, \sigma' \text{ and } G)$  is about 10 times smaller than  $B(J/\psi \rightarrow \gamma\eta') \approx 4 \cdot 10^{-3}$ . These are typical values of gluonia widths and production rates [14]. The absence of the  $\sigma$  in  $\gamma\gamma$  scattering and its presence in  $J/\Psi$  radiative decays [18] are a strong indication of its large gluon component.

### 5. UNMIXED SCALAR QUARKONIA

#### 5.1. The $a_0(980)$

The  $a_0(980)$  is the most natural and economical meson candidate associated to the divergence of the vector current:  $\partial_\mu V^\mu(x) \equiv (m_u - m_d)\bar{u}(i\gamma_5)d$ . Previous different sum rule analysis of the associated two-point correlator gives [9]:  $M_{a_0} \simeq 1 \text{ GeV}$  and the conservative range  $f_{a_0} \simeq (0.5 - 1.6) \text{ MeV}$  ( $f_\pi = 93 \text{ MeV}$ ), in agreement with the value  $1.8 \text{ MeV}$  from a hadronic kaon tadpole mass difference approach plus a  $a_0$  dominance of the  $K\bar{K}$  form factor. A 3-point function sum rule analysis gives the widths [11,21,12]:

$$\begin{aligned} \Gamma(a_0 \rightarrow \eta\pi) &\simeq 37 \text{ MeV} , \\ \Gamma(a_0 \rightarrow \gamma\gamma) &\simeq (0.3 - 1.5) \text{ keV} , \end{aligned} \quad (15)$$

while from  $SU(3)$  symmetry, we expect to have:

$$g_{a_0 K^+ K^0} \simeq \sqrt{\frac{3}{2}} g_{a_0 \eta\pi} . \quad (16)$$

The value of this coupling to  $\bar{K}K$  reproduces present data [17,18]. Analogous sum rule analysis in the four-quark scheme [21,9] gives similar values of the masses (see also the lattice results in [22]) and hadronic couplings but implies a too small value (compared with the data) of the  $\gamma\gamma$  width [21] due to the standard QCD  $\pi^2$  loop-diagram factor suppressions. Such a factor have not been taken properly in the literature. The  $(\bar{u}u - \bar{d}d)$  quark assignement for the  $a_0(980)$  is supported by present data and alternative approaches [17–19].

#### 5.2. The isoscalar partner $S_2 \equiv \bar{u}u + \bar{d}d$ of the $a_0(980)$

Analogous analysis of the corresponding 2-point correlator gives  $M_{S_2} \approx M_{a_0}$  as expected from a good  $SU(2)$  symmetry, while using 3-point function and  $SU(3)$  relation, its widths are estimated to be [12,11]:

$$\begin{aligned} \Gamma(S_2 \rightarrow \pi^+ \pi^-) &\simeq 120 \text{ MeV} , \\ \Gamma(S_2 \rightarrow \gamma\gamma) &\simeq \frac{25}{9} \Gamma(a_0 \rightarrow \gamma\gamma) \\ &\simeq 0.7 \text{ keV} . \end{aligned} \quad (17)$$

#### 5.3. The $K_0^*(1430) \equiv \bar{d}s$ and $S_3 \equiv \bar{s}s$ states

An analysis of the  $K_0^*-a_0$  mass shift due to  $SU(3)$  breakings (strange quark mass and condensate) [9,12] fixes the mass of the  $K_0^*$  to be around  $1430 \text{ MeV}$ . If a candidate around  $900 \text{ MeV}$  is confirmed, it will then be hard to reconcile with a  $\bar{q}q$  structure. An analysis of the  $S_3$  over the  $K_0^*$  2-point functions gives [12]:

$$\begin{aligned} M_{S_3}/M_{K_0^*} &\simeq 1.03 \pm 0.02 \implies \\ M_{S_3} &\simeq 1474 \text{ MeV} , \\ f_{S_3} &\simeq (43 \pm 19) \text{ MeV} , \end{aligned} \quad (18)$$

in agreement with the lattice result [23], while the 3-point function leads to [12]:

$$\begin{aligned} \Gamma(S_3 \rightarrow K^+ K^-) &\simeq (73 \pm 27) \text{ MeV} , \\ \Gamma(S_3 \rightarrow \gamma\gamma) &\simeq 0.4 \text{ keV} . \end{aligned} \quad (19)$$

In the usual sum rule approach (absence of large violations of the OPE at the sum rule stability points), one expects a small mixing between the  $S_2$  and  $S_3$  mesons before the mixing with the gluonium  $\sigma_B$ .

#### 5.4. Radial excitations

The properties of the radial excitations cannot be obtained accurately from the sum rule approach, as they are part of the QCD continuum which effects are minimized in the analysis. However, as a crude approximation and using the sum rule results from the well-known channels ( $\rho, \dots$ ), one may expect that the value of  $\sqrt{t_c}$  can localize approximately the position of the first radial excitations. Using this result and some standard phenomenological arguments on the estimate of the couplings, one may expect [12]:

$$\begin{aligned} M_{S'_2} &\approx 1.3 \text{ GeV}, \\ \Gamma(S'_2 \rightarrow \pi^+ \pi^-) &\approx (300 \pm 150) \text{ MeV}, \\ \Gamma(S'_2 \rightarrow \gamma \gamma) &\approx (4 \pm 2) \text{ keV}, \\ M_{S'_3} &\approx 1.7 \text{ GeV}, \\ \Gamma(S'_3 \rightarrow K^+ K^-) &\approx (112 \pm 50) \text{ MeV}, \\ \Gamma(S'_3 \rightarrow \gamma \gamma) &\approx (1 \pm .5) \text{ keV}. \end{aligned} \quad (20)$$

#### 5.5. We conclude that:

- Unmixed scalar quarkonia ground states are not wide, which excludes the interpretation of the low mass broad  $\sigma$  for being an ordinary  $\bar{q}q$  state.
- There can be many states in the region around 1.3 GeV ( $\sigma'$ ,  $S_3$ ,  $S'_2$ ), which should mix non-trivially in order to give the observed  $f_0(1.37)$  and  $f_0(1.5)$  states (see next sections).
- The  $f_J(1.7)$  seen to decay mainly into  $\bar{K}K$  [18], if it is confirmed to be a  $0^{++}$  state, can be *the first radial excitation of the  $S_3 \equiv \bar{s}s$  state*, but *definitely not the pure gluonium* advocated in [23].

### 6. SCALAR MIXING-LOGY

Many scenarios have been proposed in the literature for trying to interpret this region [9],[17]–[23]. However, one first needs to clarify and to confirm the data [18] for a much better selection of these different interpretations.

#### 6.1. Mixing below 1 GeV and the nature of the $\sigma$ and $f_0(980)$

In so doing, we consider the two-component mixing scheme of the bare states ( $\sigma_B$ ,  $S_2$ ):

$$\begin{aligned} |f_0\rangle &\equiv -\sin\theta_s |\sigma_B\rangle + \cos\theta_s |S_2\rangle, \\ |\sigma\rangle &\equiv \cos\theta_s |\sigma_B\rangle + \sin\theta_s |S_2\rangle \end{aligned} \quad (21)$$

A sum rule analysis of the off-diagonal 2-point correlator [35,9,12]:

$$\begin{aligned} \psi_{q\bar{q}}^S(q^2) &\equiv i \int d^4x e^{iqx} \langle 0 | T \beta(\alpha_s) G^2(x) \\ &\times \sum_{u,d,s} m_q \bar{q}q(0) | 0 \rangle, \end{aligned} \quad (22)$$

responsible for the mass-shift of the mixed states gives a small *mass mixing angle* of about  $15^\circ$ , which has been confirmed by lattice calculations using different input for the masses [23] and from the low-energy theorems based on Ward identities of broken scale invariance [16], if one uses there the new input values [9,25] of the quark and gluon condensates. In order to have more complete discussions on the gluon content of the different states, one should also determine the *decay mixing angle*. In so doing, we use the predictions for  $\sigma_B$ ,  $S_2 \rightarrow \gamma\gamma$  obtained in the previous sections and the data  $\Gamma(f_0 \rightarrow \gamma\gamma) \approx 0.3 \text{ keV}$ . Then, we deduce a *maximal decay mixing angle* and the widths [11,12]:

$$\begin{aligned} |\theta_s| &\simeq (40 - 45)^\circ, \\ \Gamma(f_0 \rightarrow \pi^+ \pi^-) &\leq 134 \text{ MeV}, \\ g_{f_0 K^+ K^-} / g_{f_0 \pi^+ \pi^-} &\approx 2, \\ \Gamma(\sigma \rightarrow \pi^+ \pi^-) &\approx (300 - 700) \text{ MeV}, \\ \Gamma(\sigma \rightarrow \gamma\gamma) &\approx (0.2 - 0.5) \text{ keV}. \end{aligned} \quad (23)$$

The huge coupling of the  $f_0$  to  $\bar{K}K$  or  $\bar{s}s$  comes from the gluon component through the large mixing with the  $\sigma$ . For this reason, the  $f_0$  can have a large singlet component, which is also suggested by independent analysis [17,15]. Extending the previous  $J/\psi \rightarrow \gamma + X$  analysis into the case of the  $\phi$ , one obtains the *new result* within this scheme [36]:

$$Br[\phi \rightarrow \gamma + f_0(980)] \approx 1.3 \times 10^{-4}, \quad (24)$$

in good agreement with the Novosibirsk data of  $(1.93 \pm 0.46 \pm 0.5) \times 10^{-4}$ . In the same way, the large coupling of the  $f_0(980)$  to  $\bar{K}K$  could also explain its production from  $D_s \rightarrow 3\pi$  decay [37].

#### 6.2. Mixing above 1 GeV and nature of the $f_0(1.37)$ , $f_0(1.5)$ , $f_0(1.6)$ and $f_0(1.7)$

As already mentioned previously, this region is quite complicated due to the proliferation of

states. We shall give below *some selection rules* which can already eliminate some of the different schemes proposed in the literature:

- **The  $f_0(1.37)$ :** If it decays into  $\sigma\sigma \rightarrow (4\pi^0)_S$ , it signals mixings with the  $\sigma$ ,  $\sigma'$  and  $G$ .
- **The  $f_0(1.5)$ :**
  - If it decays into  $\sigma\sigma$  and  $\eta'\eta$ , this signals a gluon component.
  - If it also couples to  $\pi\pi$  and  $\bar{K}K$ , this signals a  $\bar{q}q$  component which may come from the  $S'_2$ ,  $S_3$ . Then, it can result from the  $\bar{q}q$  mixings with the  $\sigma$ ,  $\sigma'$  and  $G$ , like the  $f_0(1.37)$ .
  - If it couples weakly to  $\pi\pi$  and  $\bar{K}K$ , while the ratio of its  $\eta\eta$  and  $\eta'\eta$  is proportionnal to  $1/\sin^2\theta_P$ , then it can be an almost pure gluonium state, which can be identified with the  $G$  in Table 1 obtained in the quenched approximation (this approximation is expected to be much better at higher energies using  $1/N_c$  arguments [10,38]).
- **The  $f_0(1.7)$ :** If it decays mainly into  $\bar{K}K$  [18], it is likely the radial excitation  $S'_3$  of the  $S_3(\bar{s}s)$  state rather than a gluonium.

## 7. CONCLUSIONS

We have seen that there are increasing experimental evidences for the existence of the  $\sigma$  and other scalar mesons. QCD spectral sum rule (QSSR) and low-energy theorem (LET) provide a first analysis of their gluon and quark substructure beyond the usual effective Lagrangian approach. Present data favour a maximal quarkonium-gluonium ( $\bar{q}q$ - $gg$ ) mixing scheme for the  $\sigma$  and  $f_0(980)$  mesons. Analogous scenarios, though more complicated, are encountered for higher mass scalar states. Within the scheme, the wide  $\sigma$  and the narrow  $f_0(980)$  appear to be  $\eta'$ -like particles, which can have strong couplings to meson pairs through OZI violations. The  $a_0(980)$  is a  $\bar{u}d$  state, while its strange partner  $\bar{d}s$  is expected to be the  $K_0^*(1430)$  of PDG. More experimental tests are needed for selecting different phenomenological schemes. We wish that in the near future our understanding of the scalar mesons will be improved further.

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